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Matrix String Theory on pp-waves

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Abstract: After a breif review on Matrix String Theory on flat backgrounds, we formulate matrix string models on different pp-wave backgrounds. This will be done both in the case of constant and variable RR background flux for certain exact string geometries. We exhibit the non–perturbative representation of string interaction and show how the eigenvalue tunneling drives the WKB expansion to give the usual perturbative string interaction also in supersymmetric pp-wave background cases.

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Overview of Matrix String Theory Matrix String Theory [1] (MST) connects perturbative string theory and M(atrix)-theory [2] by explaining how the theory of strings is included in the latter in a second quantization scheme. As known, eleven dimensional M-theory compactified on a circle $S^1_{(11)}$ is supposed to give rise to the type IIA string theory at strong coupling where D-particles are claimed to be the natural degrees of freedom describing the theory. To be able to reach a string theory in a perturbative regime, one may be able to use a strong/weak duality, namely the type IIB S-selfduality. Therefore, in order to perform such a program, one is forced to further compactify the theory on another circle $S^1_{(9)}$ to be able to reach type IIB string theory by a $T_{(9)}$-duality. Insodoing, the D-particles get mapped to D-strings whose winding in the nineth direction counts their number. Now it is possible to perform an S-selfduality of the type IIB string theory by mapping these D-strings in fundamental type IIB strings with the same winding in the nineth direction and weak coupling. Upon a final $T_{(9)}$-duality these strings get mapped to weakly coupled type IIA strings with fixed light cone momentum. Notice that we started with a M-theory picture in which $R_{(11)} = g_s l_s$ is a much bigger circle than $R_{(9)}$ – being the type IIA theory in the strong coupling regime – while we end up in an M-theory picture in which $R_{(11)} = g_s l_s$ is a much smaller than $R_{(9)}$. Actually, the chain of dualities we performed could have been obtained simply as a role-flip in M-theory between the two circles. The outcome of the above duality chain is that perturbative type IIA strings admit a dual description as wrapped type IIB D-strings once the light-cone momentum of the strings is associated to the winding number of the D-strings. Let us notice that from this point of view the overall transverse spacial eight dimensional geometry plays a passive role and that it can be left as an arbitrary string background. Anyway, the need of tractable cases tends to simplify the transverse geometry in a natural way. Therefore, we review the flat case.

The effective description of wrapped type IIB D-strings is obtained by considering the massless spectrum of the open strings stretched between them. This is given by the dimensional reduction to the D-string worldsheet of the $D = 1 + 9$ N=1 superYang-Mills theory with gauge group $U(N)$, that is the $D = 1 + 1$ N=(8,8) superYang-Mills theory with gauge group $U(N)$. The gauge theory coupling g weights the mass scale of the stretched open strings, that is the D-string tension, and therefore it is natural to set $g = \frac{1}{l_s g_s}$ from the duality chain, where g_s is the resulting type IIA perturbative string coupling. If all that is true, the superconformal fixed point of the (8,8) superYang-Mills theory with gauge group U(N) at the strong Yang-Mills coupling limit and in the large N regime has to realizes the free limit of type IIA string theory on flat background. To show that this is indeed the case, let us consider for simplicity just the bosonic part of the action first, that is

$$S_b = \frac{1}{2} \int d^2 z \left\{ D_{\bar{z}} X^I D_z X^I + \frac{1}{2g^2} |F_{z\bar{z}}|^2 - \frac{g^2}{2} \sum_{IJ} [X^I, X^J]^2 \right\}$$

where the X^I are in the 8_v of the R-symmetry $SO(8)$ group representing the unbroken transverse rotational group and are in the adjoint representation of the gauge group. The two dimensional integral is along a cylinder $S^1 \times \mathbf{R}$. By taking the strong coupling limit we see that classically , i.e. analyzing the potential $V=\frac{g^2}{2}\sum_{IJ}[X^I,X^J]^2$, the degrees of freedom surviving the strong coupling limit are the Cartan projected matrix fields such that $[X^I,X^J]=0$. The same argument applies to the gauge connection, once the gauge coupling gets rescaled in the covariant derivatives and in the gauge curvature. Therefore the X^I are simultaneusly diagonalizable by a large gauge transformation, i.e. $X^I=Ux^IU^\dagger$ where x^I is a diagonal matrix. Since the only gauge invariant concept is the spectrum of the X^Is itself, we find that the IR fixed point has to be given by a supersymmetric σ-model on the quotient $\left(\mathbf{R}^8\right)^N/S_N$. This classical extimation can be made precise by considering the full theory

$$S=S_b+S_f \quad \text{where} \quad S_f=\frac{1}{2}\int d^2z\left\{i\Theta_sD_z\Theta_s+i\Theta_cD_{\bar{z}}\Theta_c+2ig\Theta_s\Gamma^I\left[X^I,\Theta_c\right]\right\} \tag{0.1}$$

where the fermion fields Θ_s and Θ_c are in the 8_s and 8_c representations of $SO(8)$ and in the adjoint of the gauge group. As the bosons, also the fermions get projected to the Cartan in the strong coupling. Expanding the action around a generic Cartan valued field configuration, it is easy to see that in the strong coupling limit the only left over contribution to the integration along the non-Cartan degrees of freedom is a supersymmetric Gaussian path integral. This shows the quantum stability of the Cartan projection at strong coupling. and means that at strong coupling we find an IR dynamics governed by the action

$$S^\infty=\frac{1}{2}\int d^2z\left\{D_{\bar{z}}x^ID_zx^I+\frac{1}{2}|f_{z\bar{z}}|^2+i\theta_sD_z\theta_s+i\theta_cD_{\bar{z}}\theta_c\right\}$$

where all the fields are in a common Cartan subalgebra. As it is easy to see, the above action – due to the fact that the gauge connection is completly decoupled from the σ-model fields and that it does not carry any local degree of freedom – is equal in form to the GS type IIA action, but its field content is a bit different being given by a symmetrized sum of N copies of it, that is an orbifold σ-model on $\left(\mathbf{R}^8\right)^N/S_N$. This is because the large gauge transformation U relating the single valued field X^I and the Cartan projected x^I as $X^I=Ux^IU^\dagger$ is not required to be periodic, namely upon a 2π shift in the space circle one has $U \quad\rightarrow\quad U\cdot g$ and $x^I \quad\rightarrow\quad g^\dagger\cdot x^I\cdot g$ where $g\in S_N$. To disentangle the orbifold structure, we fix the conjugacy classes of each permutation $g\in S_N$ in irreducible cyclic ones as

$$(g)=(1)^{n_1}\cdot(2)^{n_2}\cdot\ldots\cdot(a)^{n_a}\cdot\ldots\cdot(N)^{n_N} \quad \text{where} \quad \sum_a an_a=N \tag{0.2}$$

Let us focus on a single cyclic permutation (a). It acts on the relative eigenvalues of the Cartan fields as $x^I_\alpha(\sigma+2\pi)=x^I_{\alpha+1}(\sigma)$ where $\alpha\in[1,\ldots,a]$. Considering the collection of the x^I_αs, we can collect them in a single field $\hat{x}^I$ with period $2a\pi$. Therefore the field content of the sector corresponding to the generic permutation (0.2) is given by a set of strings (one for each cyclic elementary permutation) of length a. In the large N limit, this is the representation of the second quantized GS type IIA string in the free limit, where the length parameter is mapped to the light-cone string momentum [2].

[2] See [3] for an extensive review of this point.

As far as the string interaction is concerned, there are two different appreaches. The first, which has been considered in [1], is a constructive one and goes as follows. One considers the effect of switching back perturbatively the gauge coupling. This should result as an additional vertex $S^{\infty} \quad \rightarrow \quad S = S^{\infty} + \frac{1}{g}\int d^2 z V^{(3/2,3/2)} + \ldots$ driving the orbifold CFT out of the conformal point. By S_N and $SO(8)$ symmetries and locality arguments the vertex $V^{(3/2,3/2)}$ can be fixed uniquely to be given in terms of spin operators. The vertex indeed reproduces the Mandelstam vertex insertion at the branching points of a typical string diagram.

A second approach to string interactions, which has been considered in [4], starts by looking for the string interaction in the very structure of the gauge theory. The first observation is that different long string configurations corresponds to topologically dinstinct vacua of the gauge theory. The possibility therefore arises that some instantonic effect takes place by tunneling between these different vacua. This is indeed the case. Non trivial instantons (preserving 1/2 of the original supersymmetry) are given by field configurations satisfying the Hitchin system [5]

$$F_{z\bar{z}} + g^2[\sigma, \bar{\sigma}] = 0 \quad \text{and} \quad D_z \sigma = 0 \tag{0.3}$$

where $\sigma = X^1 + iX^2$ and the other scalar fields are passive. The spectral data classifying its solutions is given by the moduli space $\mathcal{H}_N$ of holomorphic plane curves of rank N. The generic curve $\mathcal{S}$ is defined by the σ-spectral equation

$$0 = \det(\sigma(z) - \sigma \mathbf{1}_N)$$

for any solution of (0.3). The way in which these spectral curves represent the Mandelstam diagrams is discussed in detail in [4]. The branching points of the curve $\mathcal{S}$ where various sheets come together are therefore associated to the joining/splitting of strings. The gauge curvature is generically turned on only in a region of size g^{-1} around the branching points of the curve $\mathcal{S}$ where various sheets come together, while away from these points it is small corresponding to the stability of the intermediate free string configurations where σ is a normal matrix. This string interpretation of the Hitchin system allows then to make manifest the eigenvalue tunneling responsable for the string interaction and to build it directly from the gauge theory. Infact one can refine the strong coupling limit by calculating the WKB approximation of a generic matrix string amplitude by expanding the action around the solutions of the Hitchin system. Once a particular solution, i.e. a spectral curve $\mathcal{S}$, is choosen, then it singles out a precise Cartan subalgebra pattern almost everywhere and the fields eigenvalues now become all together the corresponding light-cone string coordinate fields on the spectral curve itself.

Therefore we obtain a representation for the partition sum as

$$Z_N = \sum_{\mathcal{S} \in \mathcal{H}_N} \int D[x,\theta] e^{-S(x,\theta)_{\mathcal{S}}} \times \int D[A] e^{-S(A)_{\mathcal{S}}}$$

where $S(x,\theta)_{\mathcal{S}}$ is the type IIA GS string action on $\mathcal{S}$ and $S(A)_{\mathcal{S}}$ is the action of a decoupled U(1) gauge theory on $\mathcal{S}$. The integration over the gauge field is trivial and gives

$$\int D[A] e^{-S(A)_{\mathcal{S}}} = \frac{Det' \Delta_0}{Det' \Delta_1} g^{\P_0 - \P_1}$$

where Δ_i are the Laplacian for i-differentials on $\mathcal{S}$ and $\P_i$ is the number of their zero modes. By using the Riemann-Roch formula $\P_0 - \P_1 = \chi_{\mathcal{S}}$ we can rewrite the matrix string partition sum as [3]

$$Z_N = \sum_{\mathcal{S} \in \mathcal{H}_N} g_s^{-\chi_{\mathcal{S}}} \int D[x,\theta] e^{-S(x,\theta)_{\mathcal{S}}} \tag{0.4}$$

where we identify (after the proper introduction of dimensionfull constants) $g = \frac{1}{l_s g_s}$. Eq.(0.4) shows that the two dimensional gauge theory defining matrix string theory admits an asymptotic expansion as a GS string theory already at finite N. To match the usual GS partition function we have to make sure that the moduli space of the Hitchin systems $\mathcal{H}_N$ recostructs the full moduli space of Riemann surfaces. This can be shown to happen exactly in the large N limit. As far as the interpretation of the resulting measure as the Weil-Petersson in the large N limit, see [6].

This way, the interacting structure of perturbative string theory, namely the genus expansion, is recovered as an asymptotic expansion of the gauge theory partition function around the conformal fixed point in the inverse gauge coupling which is then, accordingly with S-duality, interpreted as the string coupling.

Strings and pp-waves While it is still an open issue the quantization of string theory in curved backgrounds and the study of their finiteness properties, it appeared recently an interesting class of (2,2) pp-wave solutions of type IIB [7] generalizing the one studied in [8]. These string theories have been shown to admit a supercovariant formulation in [9] and have been shown to be exact (finite) in [9, 10]. These (2,2) solutions have been studied in the $SU(4) \times U(1)$ formalism [11]. Within this framework the type IIB GS action on a flat background is written in terms of four complex chiral (2,2) superfields X^{+l}, where $l = 1, \ldots, 4$, as

$$S_0 = \int d^2z d^4\theta X^{+l} X^{-l}$$

where the only part of the original $SO(8)$ symmetry which remains manifest is a $SU(4) \times U(1)$ one. The action relative to these new pp-wave backgrounds is written as the σ-model action

$$S_{pp}^{IIB} = \int d^2z \left\{ \int d^4\theta X^{+l} X^{-l} + \int d^2\theta W(X^{+l}) + c.c. \right\}$$

where W is the prepotential, i.e. an holomorphic function in the four chiral fuperfields. The pp-wave metric and the RR-field curvature are parametrized by this holomorphic function as $ds^2_{(10)} = -2dx^+dx^- - |\partial W|^2 (dx^+)^2 + 2dx^l d\bar{x}^l$ and the RR-fields $F^{(5)} \sim \partial^2 W$. Notice that from this exact superconformal formulation, upon a change of variables, the R-NS formulation can be obtained where the string interaction is well defined, being the background non dilatonic, as the usual genus expansion. Particularly, if $W(x) = \mu \sum_l x_l^2$, then we obtain the maximal supersymmetric pp-wave with manifest $SO(4)$ symmetry and

[3] We redefine the measure on $\mathcal{H}_N$ to include also $\frac{Det'\Delta_0}{Det'\Delta_1}$.

constant RR-flux $F^{(5)} \sim \mu$ while in general if W is a generic quadratic function, these backgrounds have constant RR-flux and less isometries.

To formulate a Matrix String Theory for such a kind of backgrounds (in order to embed them in M-theory) we have to consider their type IIA counteparts. Because of the different chirality assignment between left and right moving fermions, the analogous type IIA (2,2) σ-model has just three chiral superfields ϕ^i, where $i = 1, 2, 3$, and a twisted chiral [4] one that we call Σ. This is the type IIA counterpart of the above superfields formalism in which only an $SU(3) \times U(1)$ symmetry is manifest. The type IIA action is then given by

$$S^{IIA}_{pp} = \int d^2z \left\{ \int d^4\theta \left\{ -\frac{1}{4}\Sigma\bar{\Sigma} + \phi^i\bar{\phi}^i \right\} + \int d^2\theta W(\phi^i) + c.c. \right\} \tag{0.5}$$

where the prepotential depends now only upon the three chiral superfields. In principle one could also consider more general σ-model actions including also generalized FI terms like $\int d^2z \left\{ \int d\theta^+ d\bar{\theta}^- f(\Sigma) + c.c. \right\}$, where f is holomorphic, but we don't do it here. Notice that, in particular, if $W = 0$, this action reproduces the type IIA GS action on a flat background. Now the possibility of lifting to M-theory (via a MST picture) these backgrounds can be posed clearly.

Matrix Strings on pp-waves with constant RR flux As far as the formulation of MSTs on pp-waves with *constant* RR-flux backgrounds, the procedure to obtain the most general kind of model is to consider a generic deformation of the MST action (0.1) which is quadratic in the fermions. This is because we already know that the σ-model remains quadratic in the fermion fields. This procedure can be best studied in the ten dimensional formalism where the fermion fields Θ_s and Θ_c are collected in a single Weyl-Majorana fermion we call Ψ. Infact type IIA Matrix String Theory can be obtained by reducing the $\mathcal{N} = 1$ D=10 SYM theory with gauge group $U(N)$ down to two dimensions and we can therefore work directly in the ten dimensional theory to calculate the constraints on the possible quadratic deformations and then dimensionally reduce. This procedure has been worked out in [13]. Summarizing, we deform the $\mathcal{N} = 1$ D=10 SYM action

$$S_{sym} \quad \rightarrow \quad S_{pp} = S_{sym} + \int i\bar{\Psi}H\Psi + S_b$$

where H is a constant bi-spinor whose generic structure is $H = h_{\mu\nu\rho}\Gamma^{\mu\nu\rho}\left(\frac{1+\Gamma^{11}}{2}\right)$ and S_b is a purely bosonic action term. This has to be fixed by the requirement that a shifted supersymmetry

$$\delta A_\mu = \delta_{sym} A_\mu \quad \delta\Psi = \delta_{sym}\Psi + K[A]\epsilon$$

leaves invariant the shifted action. This is possible if $K[A]$ is a linear functional in the gauge field A and if the supersymmetry parameters ϵ satisfy certain linear differential equations. The action for the general matrix string theory model on pp-waves with

[4]For definitions, properties and notation here and in the subsequent part of this letter, we refer the reader to [12].

constant RR flux can be obtained by dimensional reduction to two dimensions of the above construction. The details and the results of this approach can be found in [13] and will not be given here.

Matrix Strings on (2,2) pp-waves with non-constant RR flux This second case is the implementation in the Matrix String Theory scheme of the exact (2,2) pp-wave backgrounds reviewed in the last section [14]. In order to treat this case, we preliminarly write down the MST action on flat background in the (2,2) superfield formalism. It reads as

$$\int d^2z d^4\theta \left(-\frac{1}{4g^2}\Sigma\bar{\Sigma} + \Phi^i\bar{\Phi}^i\right) + \int d^2z \left[d^2\theta L(\Phi^i) + c.c.\right] \tag{0.6}$$

with the prepotential $L = g\Phi^1\left[\Phi^2, \Phi^3\right]$. Here Φ^i are three chiral superfields in the adjoint representation of the gauge group $U(N)$ while Σ is the twisted chiral superfield obtained as the covariant superfield curvature. The trace over the gauge group and the covariantization of the $\Phi\bar{\Phi}$ term are understood in (0.6). Notice that in the (2,2) manifest formalism only a $U(1) \times SU(3)$ subgroup of the original $SO(8)$ R-symmetry is manifest (exactly as it was in the previous section for the action (0.5) in the flat case.).

Now, to include the σ-model prepotential W, we add a further prepotential term in (0.6) as

$$S = \int d^2z \left\{\int d^4\theta \left\{-\frac{1}{4g^2}\Sigma\bar{\Sigma} + \Phi^i\bar{\Phi}^i\right\} + \int d^2\theta \left(L(\Phi^i) + \tilde{W}(\Phi^i)\right) + c.c.\right\} \tag{0.7}$$

As far as the definitions of the matrix function $\tilde{W}$ the natural requirement is that once evaluated on Cartan fields it reproduces the prepotential in (0.5) as $\mathrm{Tr}\tilde{\mathrm{W}}\left(\Phi^{\mathrm{t}\,\mathrm{i}}\right) = \sum_{\mathrm{m}} \mathrm{W}\left({\Phi_{\mathrm{m}}}^{\mathrm{i}}\right)$ in an ortonormal basis of t. This requirement specifies these structure function up to matrix ordering. The natural ordering is of course the total symmetrization. In the related context of D-geometry this problem has been clarified [15] by showing that if the background satisfies the string equations, then the total symmetric ordering is the correct one in order to reproduce the correct open string masses assignments. Since the string backgrounds we are considering are exact, TS-duality with the type IIB D-string picture justifies this ordering. Moreover, there's a second argument in favour of the symmetric ordering which is related to the symmetries of the background. Our type IIA generic background is expilicitly invariant under $SO(2)$ (acting on the σ complex plane) and the $SU(3)$ transformations under which W is invariant (up to additional constants). Let us notice that, since the additional prepotential $L = g\Phi^1[\Phi^2, \Phi^3]$ is fully $SU(3)$ invariant and since the total symmetrization is the only matrix ordering prescription which commutes with linear transformations of the arguments, then this prescription is the only one which preserves the above background symmetries. Let us notice that this agrees with the issue raised in [16] where the symmetry of the background pp-wave metric is token as a guiding principle for the construction of a well defined string perturbation theory. We consider these arguments exaustive of a discussion about the matrix ordering prescription.

For completeness, we give the bosonic part of the action (0.7) that is

$$S_b = \frac{1}{2}\int d^2z \left\{ D_{\bar z}\phi^i D_z\bar{\phi}^{\bar i} + D_z\phi^i D_{\bar z}\bar{\phi}^{\bar i} + D_{\bar z}\sigma D_z\bar\sigma + D_z\sigma D_{\bar z}\bar\sigma + \right.$$

$$\left. \bar{F}^{\bar i}F^i + \frac{1}{2g^2}|F_{z\bar z}|^2 - \frac{g^2}{2}[\sigma,\bar\sigma]^2 - \frac{g^2}{2}[\phi^i,\bar{\phi}^{\bar i}]^2 + g^2[\bar\sigma,\phi^i][\sigma,\bar{\phi}^{\bar i}] + g^2[\sigma,\phi^i][\bar\sigma,\bar{\phi}^{\bar i}] \right\}$$

where $\bar{F}^i = \frac{\partial(\tilde{W}+L)}{\partial\Phi^i}(\phi)$.

A first check of our model is that in the strong coupling it has to reduce to a symmetrized orbifold of the type IIA GS action (0.5). This is indeed the case, since the additional prepotential $\tilde W$ is independent upon the gauge coupling and therefore, assuming that the RR flux is weak with respect to the gauge coupling g, it does not interfere with the strong coupling limit procedure as described for the flat case in the first section.

A second important point is to check if the WKB expantion of the matrix string partition function still reproduces the perturbative expansion of string theory in a way similar to the one we revied in the first section. This can be again shown to happen in this case (the case of generic constant RR flux [13] seems to be more difficult because of lack of control on the supersymmetric instantons if any). Actually, the possible BPS field configurations for the MST on pp-wave we are studying are of two types.

- Static: these are 1/2 BPS solutions where the σ field is passive, the gauge connection is flat and the ϕ^is satisfy the equations

$$D_0\phi^i = 0 \quad D_1\phi^i + F^i = 0 \quad \sum_i \left[\phi^i, \bar{\phi}^{\bar i}\right] = 0$$

- Instanton: again 1/2 BPS solutions where the ϕ^is are passive and σ and the gauge connection satisfy again the Hitchin equations (0.3).

Since F-term/derivative-terms saturation implies staticity, there does not exist other istanton type solutions but the ones given by the Hitchin equations. This implies that, as in the flat case, these drive a tunneling between the different possible long string configurations of the orbifold σ-model found at strong coupling. The calculation of the WKB expansion of the partition function goes exactly like in the flat case as well as the evaluation of the gauge connection determinants.

Conclusions and open questions It sounds very much like that matrix string theory should find its proper place in gauge/string theory duality picture [17]. How and if it will happen is a nice challenging question we are facing from this point of view.

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References

[1] L. Motl, [hep-th 9701025]. T. Banks and N. Seiberg, Nucl. Phys. **B 497**, p. 41; (1997) [hep-th 9702187]. R. Dijkgraaf, E. Verlinde, H. Verlinde, Nucl. Phys. **B 500**, p. 43; (1997) [hep-th 9703030].

[2] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. **D 55**, p. 5112; (1997) [hep-th 9610043].

[3] R. Dijkgraaf, arXiv:hep-th/9912104.

[4] G. Bonelli, L. Bonora and F. Nesti, Phys. Lett. **B 435**, p. 303; (1998) [hep-th 9805071]; G. Bonelli, L. Bonora and F. Nesti, Nucl. Phys. **B 538**, p. 100; (1999) [hep-th 9807232]; G. Bonelli, In Corfu 1998, Quantum aspects of gauge theories, supersymmetry and unification 323-332 [hep-th 9812120]; G. Bonelli, L. Bonora, F. Nesti and A. Tomasiello, Nucl. Phys. **B 554**, p. 103; (1999) [hep-th 9901093]; G. Bonelli, L. Bonora, F. Nesti and A. Tomasiello, Nucl. Phys. **B**564 (2000) 86-102 [hep-th 9905092]

[5] N. J. Hitchin, Proc. Lond. Math. Soc. **55** (1987) 59.

[6] T. Wynter, Nucl. Phys. B **580** (2000) 147 [arXiv:hep-th/9905087]. P. Brax, JHEP **0007** (2000) 020 [arXiv:hep-th/9912103].

[7] J. Maldacena and L. Maoz, arXiv:hep-th/0207284.

[8] R. R. Metsaev, Nucl. Phys. B **625** (2002) 70 [arXiv:hep-th/0112044].

[9] N. Berkovits and J. Maldacena, arXiv:hep-th/0208092.

[10] J. G. Russo and A. A. Tseytlin, arXiv:hep-th/0208114.

[11] M.B. Green and J.H. Schwarz, Nucl. Phys. B **181** 502.

[12] E. Witten, arXiv:hep-th/9312104 (section [4.1]) and E. Witten, Nucl. Phys. B **403** (1993) 159 [arXiv:hep-th/9301042] (section 2).

[13] G. Bonelli JHEP **0208** (2002) 022 [arXiv:hep-th/0205213].

[14] G. Bonelli, arXiv:hep-th/0209225.

[15] M. R. Douglas, Adv. Theor. Math. Phys. **1** (1998) 198 [arXiv:hep-th/9703056]. M. R. Douglas, Nucl. Phys. Proc. Suppl. **68** (1998) 381 [arXiv:hep-th/9707228]. M. R. Douglas, A. Kato and H. Ooguri, Adv. Theor. Math. Phys. **1** (1998) 237 [arXiv:hep-th/9708012].

[16] C. S. Chu, V. V. Khoze, M. Petrini, R. Russo and A. Tanzini, arXiv:hep-th/0208148.

[17] A.M. Polyakov, arXiv:hep-th/0110196.